

Research on Financing Strategies for Capital-Constrained Green Product Manufacturers Under Retailer Competition

Fengli Tong, Xuemei Yin, Yonglong Wang

School of Business Administration, Chongqing Technology and Business University, Chongqing, 400067, China

Abstract: This paper investigates the financing strategies of capital-constrained green product manufacturers in the presence of retailer competition. Within a green supply chain featuring manufacturer production, this paper considers that green product manufacturers can alleviate capital constraints through three financing approaches: external financing, internal financing, and mixed external-internal financing. Three game models corresponding to the financing modes are established and solved by backward induction to identify manufacturers' optimal financing strategies and relevant influencing factors. The results reveal that the manufacturer's profit dominance among different financing models depends on financing interest rates. If the interest rate of external financing is higher than that of internal financing, the manufacturer achieves the highest profit under internal financing and the lowest profit under external financing. Conversely, the manufacturer's profit reaches its maximum under external financing and minimum under internal financing. By contrast, the profit dominance of Retailer 1 and Retailer 2 across financing models is associated with the product competition intensity coefficient.

Keywords: Financing strategy, capital constraint, retailer competition.

1. Introduction

Driven by the "Dual Carbon" goal, green development has become a critical direction for corporate growth. Policies such as Opinions on Accelerating the Establishment of a Legal and Policy System for Green Production and Consumption urge enterprises to strengthen their responsibilities in green management and expand the supply of green products [1]. Against this backdrop, numerous enterprises have actively pursued green transformation. For instance, CATL devotes substantial efforts to developing green battery technologies and promotes the advancement of the new energy vehicle industry.

In practice, the production of green products usually requires substantial capital investment, which leaves some enterprises trapped by capital constraints and unable to invest in green technologies [2]. Financing serves as an effective tool to ease corporate capital pressure and is widely adopted in practice. Common financing modes include external financing, internal financing, and mixed external-internal financing. For example, Alibaba once obtained external financing from SoftBank Group, Vanguard Group and other institutions. Capital-abundant firms such as Walmart and Marks & Spencer have provided internal financing for green production projects of their upstream manufacturers [3]. The upstream suppliers of TCL Group acquired external financing via JD Technology's "Supply Loan", and received internal capital support from nearly one hundred downstream air-conditioning dealers through purchase financing schemes, facilitating product sales and capital recovery.

Accordingly, this chapter focuses on a supply chain system consisting of one capital-constrained green product manufacturer and two retailers, and investigates the optimal financing strategies for the capital-constrained green product manufacturer under retailer competition. Specifically, this chapter addresses the following three research questions: What is the optimal financing strategy for capital-constrained

green product enterprises to maximize their profits? Which factors affect the financing strategy selection of green product enterprises? How should supply chain members make optimal decisions on pricing and green technology investment? To answer these questions, this chapter constructs game models for external financing, internal financing, and mixed external-internal financing, and derives their analytical equilibrium solutions. Comparative analysis and numerical analysis are conducted to identify the optimal financing strategy of the capital-constrained green product manufacturer.

2. Problem Description and Assumptions

2.1. Problem Description

This chapter considers a two-level supply chain system composed of a green product manufacturer and two competing retailers (R1 and R2). The manufacturer determines green technology investment and production decisions, and supplies products to retailers through distribution channels. The two retailers sell green products to the end market in a competitive setting. Nevertheless, the manufacturer faces capital constraints when investing in green technologies and may resort to external financing (BF) from third-party financial institutions or internal financing (IF) provided by retailers. Against this backdrop, this paper focuses on the manufacturer's selection of financing strategies.

Three models are developed to investigate the above research problem:

- ① BB Model (pure external financing game model): The manufacturer adopts pure external financing and obtains funds solely from third-party financial institutions, while neither retailer participates in internal financing.
- ② BR Model (pure internal financing game model): The manufacturer uses pure internal financing and acquires financial support from R1 and R2.

③ BM Model (mixed external-internal financing game model): The manufacturer implements mixed external-internal financing strategy, namely combining external financing with unilateral internal financing, where internal financing is obtained from R2.

2.2. Model Assumptions

To facilitate quantitative analysis, the following basic assumptions are proposed:

Assumption 1: Market demand is affected by both retail price and the level of green technology investment. The demand function is expressed as:

$$\begin{cases} d_1 = a - p_1 + \lambda p_2 + e_1 - \tau e_2 \\ d_2 = a - p_2 + \lambda p_1 + e_2 - \tau e_1 \end{cases} \quad (1)$$

Consumers in the market have homogeneous preferences for low-carbon green products, and the two products compete in the marketplace. Let a denote the potential market demand, p_1 and p_2 denote the selling prices of the two products, and λ denote the competition intensity coefficient between the two products. A larger λ represents fiercer competition and stronger product substitutability. τ

represents the green sensitivity coefficient of demand. Lower product prices, higher prices of competing products, and higher green technology investment exert positive effects on market demand.

Assumption 2: The manufacturer possesses initial capital for producing conventional products but encounters capital constraints when producing green products. In this case, the manufacturer can apply for external financing from third-party financial institutions. Meanwhile, capital-abundant retailers can provide internal financing to the manufacturer. Assume that the cost of green technology investment is $C(e) = ke^2$, where e is the green technology investment level and k is the cost coefficient of green technology investment.

Assumption 3: The manufacturer can choose external financing, internal financing, or mixed external-internal financing based on profit maximization. The competition between Retailer 1 and Retailer 2 follows Nash equilibrium. The manufacturer and the two retailers engage in Stackelberg game, where the manufacturer acts as the game leader.

Notations and their definitions used in this chapter are summarized in Table 1.

Table 1. Notations and definitions

Notation	Definition	Notation	Definition
p_i	Retail price of products sold by Retailer i	τ	Ratio of the green sensitivity coefficient of demand
e_i	Green technology investment level of Retailer i 's product	$r_\omega (\omega \in 1, 2)$	{External financing interest rate, Internal financing interest rate}
d_i	Demand quantity for Retailer i	π_M^j	Manufacturer's profit under Model j
w_i	Wholesale price of products for retailers i	π_{Ri}^j	Retailer i 's profit under model j
k	Cost coefficient of green technology investment	BB	External financing model
a	Market demand scale	BR	Internal financing model
λ	Product competition intensity coefficient	BM	Mixed external-internal financing model
$i \in \{1, 2\}$	{Retailer 1, Retailer 2}		$j \in \{BB, BR, BM\}$

3. Game Model Construction and Solution

3.1. External Financing Game Model (BB Model)

Under the BB model, the manufacturer adopts only external financing when confronted with capital constraints. The specific decision sequence in this model is described as follows: (1) As the leader, the manufacturer first determines

the wholesale price and green technology investment level; (2) As followers, the two retailers simultaneously decide their retail prices and order quantities respectively; (3) The capital-constrained manufacturer borrows loans from external institutions such as banks; (4) The manufacturer uses the loan to produce green products and sells them to the two retailers separately; (5) The two retailers sell green products to consumers; (6) The manufacturer repays the principal and interest of the loan.

The profit functions of the manufacturer and retailers under the pure external financing model are given by:

$$\begin{cases} \pi_M^{BB}(e_1, w_1, e_2, w_2) = (w_1 - c)d_1 + (w_2 - c)d_2 - (1 + r_1)k[(e_1)^2 + (e_2)^2] \\ = (w_1 - c)(a - p_1 + \lambda p_2 + e_1 - \tau e_2) + (w_2 - c)(a - p_2 + \lambda p_1 + e_2 - \tau e_1) - (1 + r_1)k[(e_1)^2 + (e_2)^2] \\ \pi_{R1}^{BB}(p_1) = (p_1 - w_1)d_1 \\ = (p_1 - w_1)(a - p_1 + \lambda p_2 + e_1 - \tau e_2) \\ \pi_{R2}^{BB}(p_2) = (p_2 - w_2)d_2 \\ = (p_2 - w_2)(a - p_2 + \lambda p_1 + e_2 - \tau e_1) \end{cases} \quad (2)$$

Backward induction is applied to derive the equilibrium wholesale price, green technology investment level, retail price and profits of the manufacturer and retailers.

Accordingly, Proposition 1 can be obtained.

Proposition 1 Under the BB model, the equilibrium decisions of the manufacturer and retailers are:

$$\begin{cases} w_1^{BB*} = w_2^{BB*} = \frac{2k(1+r_1)(\lambda-2)(a+c-c\lambda)+c(\tau-1)^2}{(\tau-1)^2-4k(1+r_1)(\lambda-2)(\lambda-1)} \\ e_1^{BB*} = e_2^{BB*} = \frac{(a-c+c\lambda)(\tau-1)}{(\tau-1)^2-4k(1+r_1)(\lambda-2)(\lambda-1)} \\ p_1^{BB*} = p_2^{BB*} = -\frac{2k(1+r_1)[a(3-2\lambda)-c(\lambda-1)]-c(\tau-1)^2}{(\tau-1)^2-4k(1+r_1)(\lambda-2)(\lambda-1)} \end{cases} \quad (3)$$

The equilibrium profits of the retailer and the manufacturer are expressed as:

$$\begin{cases} \pi_M^{BB*} = -\frac{2k(1+r_1)(a-c+c\lambda)^2}{(\tau-1)^2-4k(1+r_1)(\lambda-2)(\lambda-1)} \\ \pi_{R1}^{BB*} = \pi_{R2}^{BB*} = \frac{4k^2(1+r_1)^2(a-c+c\lambda)^2(1-\lambda)^2}{[(\tau-1)^2-4k(1+r_1)(\lambda-2)(\lambda-1)]^2} \end{cases} \quad (4)$$

3.2. Internal Financing Game Model (BR Model)

Under the BR model, the manufacturer adopts only the internal financing mode when facing capital constraints. In this model, if the manufacturer suffers from capital constraints, it chooses to obtain internal financing from two retailers R1 and R2, with the financing interest rate denoted by r_2 . The profit functions of all supply chain members are given by:

$$\begin{cases} \pi_M^{BR}(e_1, w_1, e_2, w_2) = (w_1 - c)d_1 + (w_2 - c)d_2 - (1+r_2)k[(e_1)^2 + (e_2)^2] \\ = (w_1 - c)(a - p_1 + \lambda p_2 + e_1 - \tau e_2) + (w_2 - c)(a - p_2 + \lambda p_1 + e_2 - \tau e_1) \\ - (1+r_2)k[(e_1)^2 + (e_2)^2] \\ \pi_{R1}^{BR}(p_1) = (p_1 - w_1)d_1 + kr_2(e_1)^2 \\ = (p_1 - w_1)(a - p_1 + \lambda p_2 + e_1 - \tau e_2) + kr_2(e_1)^2 \\ \pi_{R2}^{BR}(p_2) = (p_2 - w_2)d_2 + kr_2(e_2)^2 \\ = (p_2 - w_2)(a - p_2 + \lambda p_1 + e_2 - \tau e_1) + kr_2(e_2)^2 \end{cases} \quad (5)$$

Backward induction is used to derive the equilibrium wholesale price, green technology investment level, retail price and profits of the manufacturer and retailers.

Accordingly, Proposition 2 can be obtained.

Proposition 2 Under the BR model, the equilibrium decisions of the manufacturer and retailers are:

$$\begin{cases} w_1^{BR*} = w_2^{BR*} = \frac{2k(1+r_2)(\lambda-2)(a+c-c\lambda)+c(\tau-1)^2}{(\tau-1)^2-4k(1+r_2)(\lambda-2)(\lambda-1)} \\ e_1^{BR*} = e_2^{BR*} = \frac{(a-c+c\lambda)(\tau-1)}{(\tau-1)^2-4k(1+r_2)(\lambda-2)(\lambda-1)} \\ p_1^{BR*} = p_2^{BR*} = -\frac{2k(1+r_2)[a(3-2\lambda)-c(\lambda-1)]-c(\tau-1)^2}{(\tau-1)^2-4k(1+r_2)(\lambda-2)(\lambda-1)} \end{cases} \quad (6)$$

The equilibrium profits of the retailer and the manufacturer are expressed as:

$$\begin{cases} \pi_M^{BR*} = -\frac{2k(1+r_2)(a-c+c\lambda)^2}{(\tau-1)^2-4k(1+r_2)(\lambda-2)(\lambda-1)} \\ \pi_{R1}^{BR*} = \pi_{R2}^{BR*} = \frac{(a-c+c\lambda)^2[4k^2(1+r_2)^2(1-\lambda)^2+r_2(1-\tau)^2]}{[(\tau-1)^2-4k(1+r_2)(\lambda-2)(\lambda-1)]^2} \end{cases} \quad (7)$$

3.3. Mixed External-Internal Financing Game Model (BM Model)

Under the BM model, the manufacturer adopts the mixed external-internal financing mode when facing capital constraints. In the BM model, if the manufacturer is capital-constrained, on the one hand, the manufacturer obtains

external financing from third-party financial institutions (e.g., banks); on the other hand, one retailer (taking R2 as an example) provides internal financing support for the manufacturer, while the other retailer (R1) does not offer internal financing. The profit functions of all supply chain members are given by:

$$\begin{cases}
\pi_M^{BM}(e_1, w_1, e_2, w_2) = (w_1 - c)d_1 + (w_2 - c)d_2 - (1 + r_1)ke_1^2 - (1 + r_2)ke_2^2 \\
= (w_1 - c)(a - p_1 + \lambda p_2 + e_1 - \tau e_2) + (w_2 - c)(a - p_2 + \lambda p_1 + e_2 - \tau e_1) \\
- (1 + r_1)ke_1^2 - (1 + r_2)ke_2^2 \\
\pi_{R1}^{BM}(p_1) = (p_1 - w_1)d_1 \\
= (p_1 - w_1)(a - p_1 + \lambda p_2 + e_1 - \tau e_2) \\
\pi_{R2}^{BM}(p_2) = (p_2 - w_2)d_2 + kr_2(e_2)^2 \\
= (p_2 - w_2)(a - p_2 + \lambda p_1 + e_2 - \tau e_1) + kr_2(e_2)^2
\end{cases} \quad (8)$$

Backward induction is adopted to derive the equilibrium wholesale price, green technology investment level, retail price and profits of the manufacturer and retailers.

Accordingly, Proposition 3 can be obtained.

Proposition 3 Under the BM model, the equilibrium decisions of the manufacturer and retailers are:

$$\begin{cases}
w_1^{BM*} = \frac{\left[8ck^2(1+r_1)(1+r_2)(4-5\lambda^2+\lambda^4) + 2ak(1+\tau)\{\lambda(1+r_2)-2(r_1+1) \right. \\
\left. + [\lambda(1+r_1)-2(r_2+1)]\tau\} - 2k(2+\lambda)[4ak(1+r_1)(1+r_2)(\lambda-2)(\lambda+1) + 3c\tau^2] \right. \\
\left. - 4ck(1+\lambda)(r_1\lambda+r_2\tau^2) + c\{1-2\tau^2+\tau^4+k[2(2+r_1+r_2)(2+9\lambda+\lambda^2)\tau \right. \\
\left. - 4(3+r_1+2r_2)-6\lambda-2r_2\lambda-2(3+r_2)\lambda^2-6\lambda^2\tau^2-4r_2\lambda^2\tau^2-2r_1(4+\lambda+\lambda^2)\tau^2]\} \right]}{\Lambda_3} \\
w_2^{BM*} = \frac{\left[-8ak^2(1+r_1)(1+r_2)(\lambda-2)(\lambda+1)(\lambda+2) + 8ck^2(1+r_1)(1+r_2)(4-5\lambda^2+\lambda^4) \right. \\
\left. - 4ckr_1(\lambda+1)\tau^2 + 2ak(1+\tau)\{\lambda(1+r_1)-2(r_2+1) + [\lambda(1+r_2)-2(r_1+1)]\tau\} \right. \\
\left. + c(1-2\tau^2+\tau^4+k\{-4(3+2r_1+r_2)-6\lambda-2\lambda[r_1+2r_2+(3+r_1+2r_2)\lambda] \right. \\
\left. + 2(2+r_1+r_2)(2+9\lambda+\lambda^2)\tau - 12\tau^2-8r_2\tau^2-2(3+r_2)\lambda\tau^2-4r_1\lambda^2\tau^2-2(3+r_2)\lambda^2\tau^2\} \right]}{\Lambda_3} \\
e_1^{BM*} = \frac{(a-c+c\lambda)(1-\tau)[4k(1+r_2)(1+\lambda)(2+\lambda)-(1+\tau)^2]}{\Lambda_3} \\
e_2^{BM*} = \frac{(a-c+c\lambda)(1-\tau)[4k(1+r_1)(1+\lambda)(2+\lambda)-(1+\tau)^2]}{\Lambda_3} \\
p_1^{BM*} = -\frac{\left[(\lambda+2)\{(1+r_1)(1+r_2)(\lambda+1)[8ck^2(\lambda-1)+8ak^2(2\lambda-3)] + 4ckr_1\tau^2\} \right. \\
\left. + 4k\lambda(1+r_2)[c\lambda\tau^2-a(1+\tau)] - 2ak(1+\tau)[2\lambda\tau-3(1+\tau+r_1\tau)+r_1(2\lambda\tau-3)] + \right. \\
\left. c\{2\tau^2-\tau^4-1+k[10+2r_1+8r_2+10\lambda+4\lambda(\lambda+r_2)+2r_1\lambda(2\lambda+3) \right. \\
\left. - 2(2+r_1+r_2)(\lambda+3)(2\lambda+1)\tau+10\tau^2+2r_2\tau^2+2(5+3r_2)\lambda\tau^2]\} \right]}{\Lambda_3} \\
p_2^{BM*} = -\frac{\left[4ck(1+r_2)\lambda^2 + (1+r_1)(1+r_2)(\lambda+1)(\lambda+2)[8ck^2(\lambda-1)+8ak^2(2\lambda-3)] \right. \\
\left. + 4k\lambda(1+r_1)[c\lambda\tau^2-a(1+\tau)] - 2ak(1+\tau)[2\lambda\tau-3(1+\tau+r_1\tau)+r_1(2\lambda\tau-3)] \right. \\
\left. + c\{2\tau^2-\tau^4-1+k[10+8r_1+2r_2+2(5+2r_1+3r_2)\lambda-2(2+r_1+r_2)(3+\lambda)(1+2\lambda)\tau \right. \\
\left. + 10\tau^2+2r_1\tau^2+8r_2\tau^2+2(5+3r_1+2r_2)\lambda\tau^2]\} \right]}{\Lambda_3}
\end{cases} \quad (9)$$

The equilibrium profits of the retailer and the manufacturer

are expressed as:

$$\left\{ \begin{aligned}
\pi_{R1}^{BM*} &= \frac{4k^2(a-c+c\lambda)^2\{4k(1+r_1)(1+r_2)(1+\lambda)(\lambda-1)(2+\lambda) \\
&\quad + (1+\tau)[1+r_1+\tau(1+r_2)-m\lambda\tau-\lambda(1+n+\tau)]\}^2}{\Lambda_3^2} \\
\pi_{R2}^{BM*} &= (a-c+c\lambda)^2 \frac{\begin{aligned}
&64k^4(1+r_1)^2(1+r_2)^2(\lambda-1)^2(\lambda+1)^2(2+\lambda)^2 \\
&+n(\tau-1)^2(1+\tau)^4-8k(1+r_1)n(\lambda+1)(\lambda+2)(\tau^2-1)^2 \\
&-32k^3(1+r_1)(1+r_2)(1+\lambda)(\lambda-1)(2+\lambda)(1+\tau)[\lambda(1+m+\tau) \\
&+n(\lambda\tau-1)-1-(1+m)\tau]+4k^2\{n^2(\tau^2+1)^2(\lambda\tau-1)^2 \\
&+(1+\tau)^2[1+\tau+m\tau-\lambda(1+m+\tau)]^2+2n(9+12\lambda^3(\tau-1)^2+2\lambda^4(\tau-1)^2 \\
&+2m^2(\lambda+1)^2(2+\lambda)^2(\tau-1)^2+\tau[\tau(11+\tau)-13] \\
&+\lambda^2(26+29\tau^2+3\tau^3+\tau^4-51\tau)-\lambda(4\tau^3+\tau^4-23-18\tau^2+52\tau) \\
&+m[(16+24\lambda^3(\tau-1)^2+4\lambda^4(\tau-1)^2+18\tau^2+\tau^3-31\tau \\
&+\lambda^2(52+54\tau^2+\tau^3-103\tau)-\lambda(98\tau+2\tau^3+\tau^4-46\tau^2-47)]\}
\end{aligned}}{\Lambda_3^2} \\
\pi_M^{BB*} &= \frac{k(a-c+c\lambda)^2[8k(1+r_1)(1+r_2)(1+\lambda)(2+\lambda)-(2+r_1+r_2)(1+\tau)^2]}{\Lambda_3}
\end{aligned} \right. \quad (10)$$

Where $\Lambda_3 = 16k^2(1+r_1)(1+r_2)(4-5\lambda^2+\lambda^4)+(\tau^2-1)^2-4k(2+r_1+r_2)[2+\lambda^2-6\lambda\tau+(2+\lambda^2)\tau^2]$.

4. Comparative Analysis

The game equilibrium solutions of upstream and downstream supply chain firms under the BB, BR and BM models demonstrate that the optimal decisions and profits of supply chain participants differ across various financing modes. By comparing the optimal wholesale prices, green technology investment levels, retail prices and expected profits, the following propositions are derived.

Proposition 4 Comparing the wholesale prices of Retailer 1 under the BB, BR and BM models yields the following results:

- 1) When $r_1 > \max[r_1^A, r_2]$, then $w_1^{BR*} > w_1^{BB*} > w_1^{BM*}$;
- 2) When $r_2 > r_1 > r_1^A$, then $w_1^{BB*} > w_1^{BM*} > w_1^{BR*}$;
- 3) When $r_1^A > r_1 > r_2$ 时, then $w_1^{BR*} > w_1^{BM*} > w_1^{BB*}$;
- 4) When $r_1 < \min[r_1^A, r_2]$ 时, then $w_1^{BM*} > w_1^{BB*} > w_1^{BR*}$.

Where

$$r_1^A = \frac{\lambda+16k\lambda-4k\lambda^3-2\tau-16k\tau+4k\lambda^2\tau-\lambda\tau^2+2\tau^3}{4k(\lambda^2-4)(\lambda-\tau)}.$$

It can be observed from Proposition 4 that when the external financing interest rate is high, the wholesale price of Product 1 reaches its maximum under internal financing, followed sequentially by the manufacturer's pure external financing and mixed external-internal financing scenarios. When the external financing interest rate is moderate, the wholesale price of Product 1 is the highest under external financing, followed by mixed external-internal financing and internal financing. When the manufacturer's external financing interest rate is moderately low, the wholesale price of Product 1 is maximized under internal financing, followed by mixed external-internal financing and external financing. When the manufacturer's external financing interest rate is relatively low, the wholesale price of Product 1 is the highest under mixed external-internal financing, followed by internal financing and external financing. The underlying reason is that the manufacturer's production cost for Product 1 rises with the increase in the external financing interest rate. In

such circumstances, the manufacturer will raise the wholesale price. As the cost of external financing increases, the manufacturer becomes increasingly inclined to adopt internal financing, in which case w_1^{BR*} attains the highest value.

Proposition 5 Comparing the wholesale prices of Product 2 under the BB, BR and BM models yields the following results:

- 1) When $r_2 > \max[r_2^A, r_1]$, then $w_2^{BM*} > w_2^{BR*} > w_2^{BB*}$;
- 2) When $r_1 > r_2 > r_2^A$, then $w_2^{BB*} > w_2^{BM*} > w_2^{BR*}$.
- 3) When $r_2^A > r_2 > r_1$, then $w_2^{BR*} > w_2^{BM*} > w_2^{BB*}$.
- 4) When $r_2 < \min[r_2^A, r_1]$, then $w_2^{BB*} > w_2^{BR*} > w_2^{BM*}$.

Where

$$r_2^A = \frac{\lambda+16k\lambda-4k\lambda^3-2\tau-16k\tau+4k\lambda^2\tau-\lambda\tau^2+2\tau^3}{4k(\lambda^2-4)(\lambda-\tau)}.$$

Proposition 5 indicates that when the internal financing interest rate is high, the wholesale price of Product 2 is the highest under mixed external-internal financing, followed sequentially by the manufacturer's internal financing and pure external financing scenarios. When the manufacturer's internal financing interest rate is moderate, the wholesale price of Product 2 reaches its maximum under external financing, followed by mixed external-internal financing and internal financing. When the manufacturer's internal financing interest rate remains at a moderate level, the wholesale price of Product 2 is maximized under internal financing, followed by mixed external-internal financing and external financing. When the manufacturer's internal financing interest rate is relatively low, the wholesale price of Product 2 is the highest under external financing, followed by internal financing and mixed external-internal financing scenarios.

Proposition 6 Comparing the green technology investment levels of Product 1 under the BB, BR and BM models yields the following results:

- 1) When $r_1 > r_2 > r_2^B$, then $e_1^{BR*} > e_1^{BB*} > e_1^{BM*}$.
- 2) When $r_2 > \max[r_2^B, r_1]$, then $e_1^{BR*} > e_1^{BM*} > e_1^{BB*}$.

3) When $r_2 < \min[r_2^B, r_1]$, then $e_1^{BM*} > e_1^{BR*} > e_1^{BB*}$.

$$\text{Where } r_2^B = \frac{-4k(1+r_1)(4-5\lambda^2+\lambda^4)+(2-3\lambda+\lambda^2)\tau(2+\tau)+r_1(2+\lambda^2-6\lambda\tau+2\tau^2+\lambda^2\tau^2)}{2+4k(1+r_1)(4-5\lambda^2+\lambda^4)+\lambda^2(1-2\tau)-4\tau+3\lambda\tau^2}.$$

Proposition 6 shows that when the manufacturer's internal financing interest rate is high, the green technology investment level of Product 1 is the highest under internal financing, followed sequentially by mixed external-internal financing and pure external financing scenarios. When the manufacturer's internal financing interest rate is moderate, the green technology investment level of Product 1 reaches its maximum under internal financing, followed by internal financing and mixed external-internal financing. When the manufacturer's internal financing interest rate is relatively low, the green technology investment level of Product 1 is maximized under mixed external-internal financing, followed by internal financing and external financing scenarios.

$$\text{Where } r_2^B = \frac{-4k(1+r_1)(4-5\lambda^2+\lambda^4)+(2-3\lambda+\lambda^2)\tau(2+\tau)+r_1(2+\lambda^2-6\lambda\tau+2\tau^2+\lambda^2\tau^2)}{2+4k(1+r_1)(4-5\lambda^2+\lambda^4)+\lambda^2(1-2\tau)-4\tau+3\lambda\tau^2} \text{ and } \lambda^1 = \frac{3+3\tau^2-\sqrt{9-14\tau^2+9\tau^4}}{4\tau}.$$

Proposition 7 demonstrates that when the manufacturer's internal financing interest rate is high (i.e. $r_2 > \max[r_2^B, r_1]$, and the parameters satisfy $\lambda > \lambda^1$), or the manufacturer's internal financing interest rate is moderate (i.e. $r_1 > r_2 > r_2^B$, and the parameters satisfy $\lambda < \lambda^1$), the green technology investment level of Product 2 is the highest under mixed external-internal financing, followed sequentially by the manufacturer's internal financing and pure external financing scenarios. When the manufacturer's internal financing interest rate is high (i.e. $r_2 > \max[r_2^B, r_1]$, and the parameters satisfy $\lambda < \lambda^1$), or the manufacturer's internal financing interest rate is moderate (i.e. $r_1 > r_2 > r_2^B$, and the parameters satisfy $\lambda > \lambda^1$), the green technology investment level of Product 2 reaches its maximum under internal financing, followed by mixed external-internal financing and pure external financing scenarios. When the manufacturer's internal financing interest rate is moderate (i.e. $r_2^B > r_2 > r_1$, and the parameters satisfy $\lambda < \lambda^1$), or the manufacturer's internal financing interest rate is relatively low (i.e. $r_2 < \min[r_2^B, r_1]$, and the parameters satisfy $\lambda > \lambda^1$), the green technology investment level of Product 2 is maximized under internal financing, followed by pure external financing and mixed external-internal financing scenarios.

Proposition 8 Comparing the retail prices of products under the BB, BR and BM models yields the following results:

1) When $r_1 > r_2$, then $p_1^{BM*} > p_1^{BR*} > p_1^{BB*}$, $p_2^{BR*} > p_2^{BB*} > p_2^{BM*}$;

2) When $r_1 < r_2$, then $p_1^{BB*} > p_1^{BR*} > p_1^{BM*}$, $p_2^{BM*} > p_2^{BB*} > p_2^{BR*}$.

Proposition 8 demonstrates that when the manufacturer's external financing interest rate is higher than the internal financing interest rate ($r_1 > r_2$), the retail price of Product 1 is the highest under mixed external-internal financing, followed sequentially by the manufacturer's internal financing and pure external financing scenarios. The retail price of Product 2 reaches its maximum under internal financing, followed by pure external financing and mixed external-internal financing scenarios. When the manufacturer's external financing interest rate is lower than the internal financing interest rate

Proposition 7 Comparing the green technology investment levels of Product 2 under the BB, BR and BM models yields the following results:

1) When $\lambda < \lambda^1$, $r_1 > r_2 > r_2^B$ or $\lambda > \lambda^1$, $r_2 > \max[r_2^B, r_1]$, then $e_2^{BM*} > e_2^{BR*} > e_2^{BB*}$;

2) When $\lambda < \lambda^1$, $r_2 > \max[r_2^B, r_1]$ or $\lambda > \lambda^1$, $r_1 > r_2 > r_2^B$, then $e_2^{BR*} > e_2^{BM*} > e_2^{BB*}$;

3) When $\lambda < \lambda^1$, $r_2^B > r_2 > r_1$ or $\lambda > \lambda^1$, $r_2 < \min[r_2^B, r_1]$, then $e_2^{BR*} > e_2^{BB*} > e_2^{BM*}$.

($r_1 < r_2$), the retail price of Product 2 is the highest under pure external financing, followed by internal financing and mixed external-internal financing scenarios; the retail price of Product 2 is the highest under mixed external-internal financing, followed by pure external financing and internal financing scenarios.

Proposition 9 Comparing the manufacturer's profits under the BB, BR and BM models yields the following results:

1) When $r_1 > r_2$, then $\pi_M^{BR*} > \pi_M^{BM*} > \pi_M^{BB*}$;

2) When $r_1 < r_2$, then $\pi_M^{BB*} > \pi_M^{BM*} > \pi_M^{BR*}$.

Proposition 9 indicates that when the manufacturer's external financing interest rate is higher than the internal financing interest rate ($r_1 > r_2$), the manufacturer's profit is maximized under internal financing, followed sequentially by mixed external-internal financing and pure external financing scenarios. When the manufacturer's external financing interest rate is lower than the internal financing interest rate ($r_1 < r_2$), the manufacturer's profit is the highest under pure external financing, followed by mixed external-internal financing and internal financing scenarios. Since the analytical expressions for retailers' optimal profits in the three scenarios are relatively complicated, numerical simulation is adopted for further analysis.

5. Conclusions

This chapter investigates a two-echelon green supply chain system consisting of one manufacturer and two competing retailers. Three game models corresponding to external financing, internal financing, and mixed external-internal financing are established, and their equilibrium solutions are derived analytically. By comparatively analyzing the equilibrium outcomes of the three financing game models, this chapter explores the optimal financing strategies for a capital-constrained green product manufacturer. The main results are summarized as follows:

1) The ordering of retail prices of the two retailers across different financing models is closely related to financing interest rates. Specifically, when the external financing interest rate is higher than the internal financing interest rate, the retail price of Product 1 reaches its maximum under mixed external-internal financing and its minimum under pure external financing; the retail price of Product 2 is maximized

under internal financing and minimized under mixed external-internal financing. When the external financing interest rate is lower than the internal financing interest rate, the retail price of Product 1 attains the maximum under pure external financing and the minimum under mixed external-internal financing; the retail price of Product 2 is the highest under mixed external-internal financing and the lowest under internal financing. In addition, when the two interest rates are equal, the retail prices of Product 1 and Product 2 are identical under all three financing modes.

2) The dominance of the manufacturer's profit across financing models depends on financing interest rates. Specifically, if the external financing interest rate exceeds the internal financing interest rate, the manufacturer achieves the highest profit under internal financing and the lowest profit under pure external financing. Conversely, the manufacturer's profit is maximized under pure external financing and minimized under internal financing.

3) The profit dominance of Retailer 1 and Retailer 2 under different financing models is determined by financing interest rates and the product competition intensity coefficient. Specifically, Retailer 1 always obtains the highest profit under internal financing, while Retailer 2 always earns the lowest profit under pure external financing. The suboptimal choices of Retailer 1 and Retailer 2 differ with the relative magnitude between external and internal financing rates as well as varying levels of product competition intensity.

References

- [1] Yu, Y., Han, X., & Hu, G. (2016). Optimal production for manufacturers considering consumer environmental awareness and green subsidies. *International Journal of Production Economics*, 182, 397–408.
- [2] Krass, D., Nedorezov, T., & Ovchinnikov, A. (2013). Environmental taxes and the choice of green technology. *Production and Operations Management*, 22(5), 1035–1055.
- [3] Tang, T., Xu, H., & Zhang, Z. (2023). Operation decisions of an online dual-channel supply chain under different financing strategies. *Chinese Journal of Management Science*, 31(10), 106–115.
- [4] Li, S., & Chen, K. (2020). Pricing strategy of new energy vehicle embedded technical service providers under competitive environment. *Journal of Systems & Management*, 29(3), 561–572.
- [5] Cao, Y., Yi, C. Q., Wan, G. Y., et al. (2022). An analysis on the role of blockchain-based platforms in agricultural supply chains. *Transportation Research Part E: Logistics and Transportation Review*, 163, 102731.
- [6] Heydari, J., Govindan, K., & Aslani, A. (2019). Pricing and greening decisions in a three-tier dual channel supply chain. *International Journal of Production Economics*, 217, 185–196.
- [7] Heydari, J., Govindan, K., & Aslani, A. (2019). Pricing and greening decisions in a three-tier dual channel supply chain. *International Journal of Production Economics*, 217, 185–196.
- [8] Xu, J., Chen, Y., & Bai, Q. (2016). A two-echelon sustainable supply chain coordination under cap-and-trade regulation. *Journal of Cleaner Production*, 135, 42–56.