

# Supply Chain Bargaining with Flexible Disagreement Points: An Extension of the Kalai-Smorodinsky Solution

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**Abstract:** This paper extends the supply chain bargaining framework of Feng et al. [1] by introducing flexible disagreement points — a departure from the classical assumption that negotiating parties maintain fixed reservation utilities throughout negotiations. Flexibility parameters  $\tau_S \geq 0$  and  $\tau_R \geq 0$ , expressed as percentages of each party's walk-away value, enable meaningful cross-channel comparison. Analysing two canonical channel structures — a one-to-two channel (monopoly supplier, competing retailers) and a two-to-one channel (competing suppliers, monopoly retailer) — we compare the Kalai-Smorodinsky (KS) and Nash Bargaining (NB) solutions at the implied equivalence point  $\theta = 0.6$ . Our key findings are: (1) both solutions produce identical outcomes at baseline, confirming Feng et al.'s characterisation; (2) under flexibility, NB is  $1.2\times$  more reactive than KS in the one-to-two channel and  $8.5\times$  more reactive in the two-to-one channel; and (3) NB's hyper-reactivity destroys approximately 9% of total supply chain value in the two-to-one channel, while KS maintains near-baseline efficiency. These results demonstrate that the KS solution is not only "more reasonable" at baseline but also "more stable" under flexibility — and this stability has direct welfare consequences for supply chain performance.

**Keywords:** Supply chain management, Bargaining theory, Kalai-Smorodinsky solution, Nash bargaining, Flexible disagreement points, Channel coordination.

## 1. Introduction

Supply chain relationships are increasingly governed by negotiated agreements rather than posted prices or hierarchical control. When a supplier and a retailer set a wholesale price through bilateral negotiation, the outcome depends not only on what each party can earn from the deal, but also on what each party would earn if the deal falls apart. This "walk-away value" — formally known as the disagreement point — is a cornerstone of bargaining theory and plays a decisive role in determining prices, profits, and overall supply chain efficiency.

The two most prominent analytical frameworks for modelling such negotiations are the Nash Bargaining (NB) solution [2] and the Kalai-Smorodinsky (KS) solution [3]. The NB solution maximises the weighted product of the parties' gains above their disagreement points, requiring an exogenous specification of bargaining power  $\theta$ . The KS solution instead imposes proportionality: each party receives the same fraction of their maximum achievable gain, yielding a unique solution without requiring any input of bargaining power. Feng et al. [1] provide the first rigorous comparison of these two solutions in competing supply chain settings, demonstrating that KS produces more balanced outcomes and corresponds implicitly to NB with  $\theta = 0.6$ .

Despite this advance, all existing models — including Feng et al. [1] — assume that disagreement points are fixed throughout negotiation. In practice, parties frequently signal willingness to accept outcomes below their stated walk-away value. A monopolist supplier facing relationship pressures may hint at flexibility; a monopolist retailer seeking to close a deal quickly may communicate willingness to compromise. We term this phenomenon flexibility, and its consequences under different bargaining solutions have not previously been studied.

To see why this matters practically, consider the following scenario. A monopoly retailer sources products from two

competing suppliers. If the retailer signals willingness to accept lower profits — perhaps to preserve a long-term supply relationship or to respond to market pressure — what happens to the wholesale price it must pay? The answer depends critically on which bargaining framework governs the negotiation. Under NB dynamics, this signal can trigger a near-tripling of the wholesale price, destroying approximately 9% of total supply chain value. Under the KS solution, the same signal produces only a modest 20% price increase. This divergence — an 8.5-fold difference in sensitivity — is the central finding of this paper.

Our contributions are four-fold. First, we introduce a percentage-based flexibility formulation enabling cross-channel comparisons despite differences in walk-away value scale. Second, we validate computationally that KS equals NB( $\theta = 0.6$ ) at baseline across all competition levels. Third, we quantify the dramatic divergence between solutions under flexibility, revealing an  $8.5\times$  sensitivity gap in the two-to-one channel. Fourth, we document that NB's hyper-reactivity destroys supply chain value — a welfare consequence with direct practical implications for negotiation design.

The remainder of this paper proceeds as follows. Section 2 reviews related literature. Section 3 presents the model framework. Section 4 reports results. Section 5 discusses implications and limitations. Section 6 concludes.

## 2. Literature Review

### 2.1. Supply Chain Coordination and Bargaining

The misalignment of incentives between decentralised firms — producing double marginalisation — has motivated extensive research into coordination mechanisms [4]. Cachon [5] surveys contractual coordination through wholesale price contracts, revenue sharing, and quantity flexibility arrangements. The role of bilateral bargaining in distribution channels was formalised by Iyer and Villas-Boas [6], who

showed how retailer bargaining power shapes channel structure choices. Feng and Lu [7] further examine bargaining versus Stackelberg contracting, finding that the preferred mode depends on competitive intensity and cost structures.

## 2.2. Nash Bargaining and Kalai-Smorodinsky Solutions

Nash [2] established the axiomatic foundation of cooperative bargaining, proposing a solution that maximises the product of both parties' gains above their disagreement points. The asymmetric generalisation, which weights gains by bargaining power  $\theta$ , is widely applied in supply chain models [8]. Kalai and Smorodinsky [3] criticised Nash's Independence of Irrelevant Alternatives axiom as overly restrictive and proposed an alternative based on monotonicity: each party should receive the same proportion of their maximum attainable gain. Roth [9] provides a thorough comparison, noting that NB depends only on local Pareto frontier properties whereas KS accounts for the entire feasible set through utopia points. Moulin [13] shows that the KS solution can be implemented through a specific alternating-offers procedure.

## 2.3. Bargaining in Competing Supply Chains

Feng et al. [1] provide the first comprehensive comparison of KS and NB in competing supply chain settings. Their key results are: (i)  $w^{KS} = 1/5$  in the one-to-two channel and  $w^{KS} = (1 - \eta)/(5 - \eta)$  in the two-to-one channel; (ii)

the KS solution corresponds to NB with implied bargaining power  $\theta \approx 0.6$ ; and (iii) KS avoids the extreme profit distributions possible under NB with asymmetric power. Nagarajan and Sošić [10] survey cooperative game-theoretic approaches to supply chain coordination, highlighting the critical role of solution concept selection.

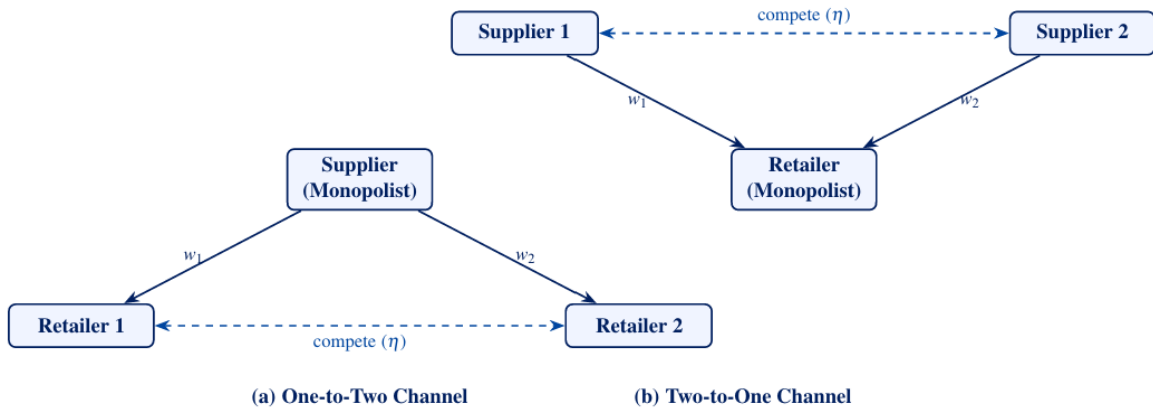
## 2.4. Disagreement Points and the Research Gap

The role of disagreement points in bargaining has received theoretical attention [11]. In supply chain contexts, disagreement points represent profits obtainable from alternative trading partners if negotiation breaks down. While Feng et al. [1] establish the advantages of KS under fixed disagreement points, no existing study examines how these advantages persist — or erode — when parties signal flexibility by voluntarily reducing their effective walk-away values. This paper addresses that gap directly.

# 3. Model Framework

## 3.1. Channel Structures

We analyse two canonical competing supply chain structures following Feng et al. [1]. In the one-to-two channel, a monopoly supplier sells to two competing retailers. In the two-to-one channel, two competing suppliers sell to a monopoly retailer. Fig. 1 illustrates both structures.



**Figure 1.** Channel structures analysed in this paper. Arrows indicate wholesale price flows; dashed lines indicate competitive interaction governed by substitutability parameter  $\eta \in (0, 1)$

## 3.2. Demand System and Profit Functions

Consumer demand follows a linear differentiated structure. Let  $p_i$  denote the retail price and  $q_i$  the quantity for product  $i$ :

$$q_i = a - b p_i + \eta(p_j - p_i), \quad i, j \in \{1, 2\}, i \neq j \quad (1)$$

The parameter  $\eta \in (0, 1)$  captures competitive intensity:  $\eta \rightarrow 0$  denotes nearly independent products;  $\eta \rightarrow 1$  denotes near-perfect substitutes. Following Feng et al. [1], we normalise  $a = b = 1$  and set marginal cost  $c = 0$ .

Given wholesale prices  $(w_1, w_2)$ , retailers compete in Cournot fashion. The symmetric equilibrium quantity is:

$$q_i^*(w_i, w_j) = \frac{(2 + \eta)(1 - w_i) - \eta(1 - w_j)}{(2 + \eta)^2 - \eta^2} \quad (2)$$

In symmetric equilibrium ( $w_1 = w_2 = w$ ), this simplifies to  $q^*(w) = (1 - w)/[2(1 + \eta)]$ . The resulting profit functions are:

$$\pi(w) = \frac{(1 + 2\eta)(1 - w)^2}{4(1 + \eta)^2} \quad (3)$$

$$\Pi(w) = \frac{w(1 - w)}{2(1 + \eta)} \quad (4)$$

## 3.3. Disagreement Points

The disagreement point specifies each party's payoff if bargaining fails. In the one-to-two channel: the monopoly supplier falls back on trading with only the remaining retailer, yielding  $D^0 = 1/8 = 0.08$ ; a retailer losing its supplier earns  $d^0 = 0$ . In the two-to-one channel: a supplier losing the only retailer earns  $D^0 = 0$ ; the monopoly retailer trades with the remaining supplier, yielding  $d^0 \approx 0.1975$  at  $\eta = 0.5$ ; more generally,  $d^0(\eta) = (1 - \eta)^2/[4(1 + \eta)(2 - \eta)^2]$ . The key asymmetry: the monopolist always has a positive walk-away value; the competing side does not.

## 3.4. Utopia Points

The utopia point  $\bar{x}_i$  is the maximum each party can

achieve when the other receives exactly their disagreement payoff. For the one-to-two channel:  $\Pi = 1/[8(1 + \eta)]$  and  $\bar{\pi} = (1 + 2\eta)/[4(1 + \eta)^2]$ . Utopia points anchor the KS solution and provide the geometric bounds that prevent explosive price responses to flexibility signals — a property analysed in Section 4.4.

### 3.5. Bargaining Solutions

The KS solution satisfies the proportionality condition — each party receives the same fraction of their maximum achievable gain:

$$\frac{\Pi(w) - D^0}{\Pi - D^0} = \frac{\pi(w) - d^0}{\pi - d^0} \quad (5)$$

This yields:  $w^{KS} = 1/5 = 0.200$  (one-to-two) and  $w^{KS} = (1 - \eta)/(5 - \eta)$  (two-to-one).

The NB solution with retailer bargaining power  $\theta \in [0,1]$  maximises:

$$\max_w [\pi(w) - d^0]^\theta [\Pi(w) - D^0]^{1-\theta} \quad (6)$$

This yields:  $w^{NB} = (1 - \theta)/2$  (one-to-two) and  $w^{NB} = (1 - \theta)(1 - \eta)/(2 - \eta + \eta\theta)$  (two-to-one).

### 3.6. Flexibility Extension

Definition 1 (Flexible Disagreement Points). Let  $\tau_S \geq 0$  and  $\tau_R \geq 0$  denote supplier and retailer flexibility parameters. The effective disagreement points are:

$$D^{eff} = D^0 - \tau_S(\text{supplier effective walk - away}) \quad (7)$$

$$d^{eff} = d^0 - \tau_R(\text{retailer effective walk - away}) \quad (8)$$

A value  $\tau = 0$  means the party requires its full outside option;  $\tau > 0$  signals willingness to accept less in order to facilitate agreement. To enable cross-channel comparison, flexibility is expressed as a *percentage of walk-away value*:

$$\tau_S = \frac{\varphi}{100} \times D^0, \quad \tau_R = \frac{\varphi}{100} \times d^0 \quad (9)$$

Where  $\varphi \in [0,30]$  is the flexibility percentage. This normalisation is critical: a raw  $\tau = 0.05$  represents 62.5% of the supplier's walk-away in the one-to-two channel but only 25% of the retailer's walk-away in the two-to-one channel. The percentage formulation gives the same economic meaning to “25% flexibility” in both channels. Table 1 presents the full conversion.

**Table 1.** Flexibility percentage to  $\tau$  conversion

Flexibility (%)	$\tau_S$ (One-to-Two, $D^0 = 0.08$ )	$\tau_R$ (Two-to-One, $d^0 = 0.1975$ )
0	0.000	0.000
10	0.008	0.020
20	0.016	0.040
25	0.020	0.049
30	0.024	0.059

An important structural observation: because the competing side's disagreement point is already zero, only the monopolist's flexibility can affect outcomes — supplier flexibility ( $\tau_S$ ) in the one-to-two channel and retailer flexibility ( $\tau_R$ ) in the two-to-one channel.

Both solutions are extended by substituting  $D^{eff}$  and  $d^{eff}$  for  $D^0$  and  $d^0$  in Eqs. (5) and (6). We focus on simultaneous bargaining and compare both solutions at  $\theta = 0.6$ , the implied equivalence point: setting  $w^{KS} = w^{NB}$  in

the one-to-two channel gives  $1/5 = (1 - \theta)/2$ , yielding  $\theta = 0.6$  (an identical result holds in the two-to-one channel). At baseline, this equivalence means both solutions produce identical outcomes, so any divergence observed under flexibility reflects purely structural differences between them.

Channel efficiency measures proximity to the integrated first-best:

$$\text{Efficiency} = \frac{\Pi(w^*) + \pi(w^*)}{\Pi^{int} + \pi^{int}} \quad (10)$$

Where the integrated benchmark is computed at  $w = c = 0$ .

## 4. Results

### 4.1. Baseline Validation

Table 2 confirms that KS and NB( $\theta = 0.6$ ) agree exactly at baseline across all channel structures and competition levels, validating our computational framework and establishing the foundation for flexibility analysis.

**Table 2.** Baseline equivalence: KS = NB( $\theta = 0.6$ ) at  $\tau = 0$

Channel	$\eta$	$w^{KS}$	$w^{NB}(\theta = 0.6)$	$\theta^{KS}$
One-to-Two	0.3	0.2000	0.2000	0.600
One-to-Two	0.5	0.2000	0.2000	0.600
One-to-Two	0.7	0.2000	0.2000	0.600
Two-to-One	0.3	0.1489	0.1489	0.600
Two-to-One	0.5	0.1111	0.1111	0.600
Two-to-One	0.7	0.0698	0.0698	0.600

In the one-to-two channel,  $w^{KS} = 0.200$  is constant across all  $\eta$  — a structural result in which substitutability terms cancel in the KS proportionality ratios. In the two-to-one channel, prices decrease with  $\eta$  following  $w^{KS} = (1 - \eta)/(5 - \eta)$ : as suppliers become more substitutable they undercut each other more aggressively, driving the wholesale price down. Both results replicate Feng et al. [1] exactly.

### 4.2. One-to-Two Channel: Supplier Flexibility Effects

When the supplier signals flexibility, their effective walk-away falls from  $D^0 = 0.08$  to  $D^{eff} = 0.08 - \tau_S$ , expanding the bargaining surplus. Both solutions respond by lowering the wholesale price. Table 3 presents results at  $\eta = 0.5$ .

**Table 3.** One-to-two channel: price sensitivity to supplier flexibility ( $\eta = 0.5$ )

Flex. (%)	$\tau_S$	$w^{KS}$	$w^{NB}$	Ratio
0	0.000	0.200	0.200	1.0×
10	0.008	0.182	0.181	1.1×
20	0.016	0.166	0.161	1.1×
25	0.020	0.161	0.153	1.2×
30	0.024	0.157	0.142	1.4×

At 25% flexibility the sensitivity ratio is:

$$\text{Ratio}_{1 \rightarrow 2} = \frac{|-23.6\%|}{|-19.7\%|} = 1.2 \times \quad (11)$$

NB is 1.2 times more reactive than KS — a modest but consistent divergence. Both solutions move in the same

direction (price falls), the supplier suffers for signalling flexibility, and retailers benefit.

### 4.3. Two-to-One Channel: Retailer Flexibility Effects

**Table 4.** Two-to-one channel: price sensitivity to retailer flexibility ( $\eta = 0.5$ )

Flex. (%)	$\tau_R$	$w^{KS}$	$w^{NB}$	KS change	Ratio
0	0.000	0.111	0.111	—	1.0×
10	0.020	0.120	0.198	+8.1%	2.2×
20	0.040	0.133	0.279	+18.0%	4.6×
25	0.049	0.133	0.297	+19.6%	8.5×
30	0.059	0.139	0.309	+25.2%	9.7×

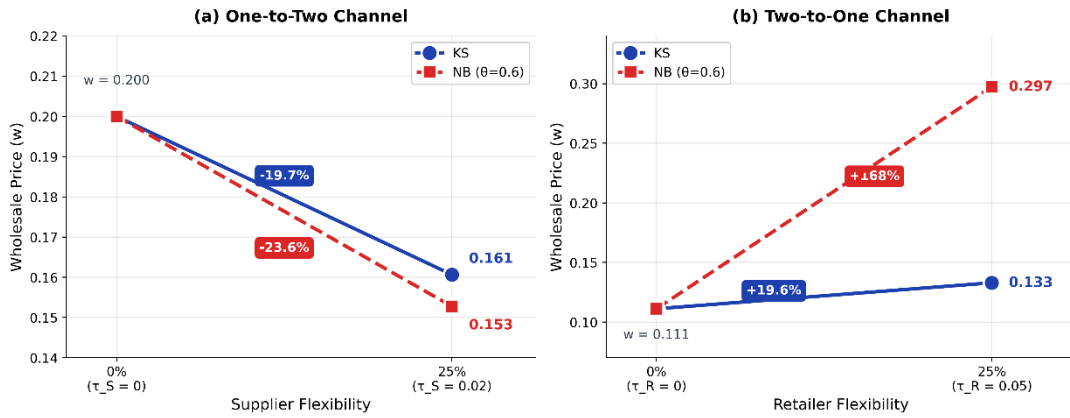
This channel produces the paper’s central finding. When the retailer signals flexibility, their effective walk-away falls from  $d^0 = 0.1975$  to  $d^{eff} = 0.1975 - \tau_R$ . Both solutions raise the wholesale price, but the magnitude of response diverges dramatically. Table 4 presents results at  $\eta = 0.5$ .

At 25% flexibility:

$$Ratio_{2 \rightarrow 1} = \frac{167.6\%}{19.6\%} = 8.5 \times \quad (12)$$

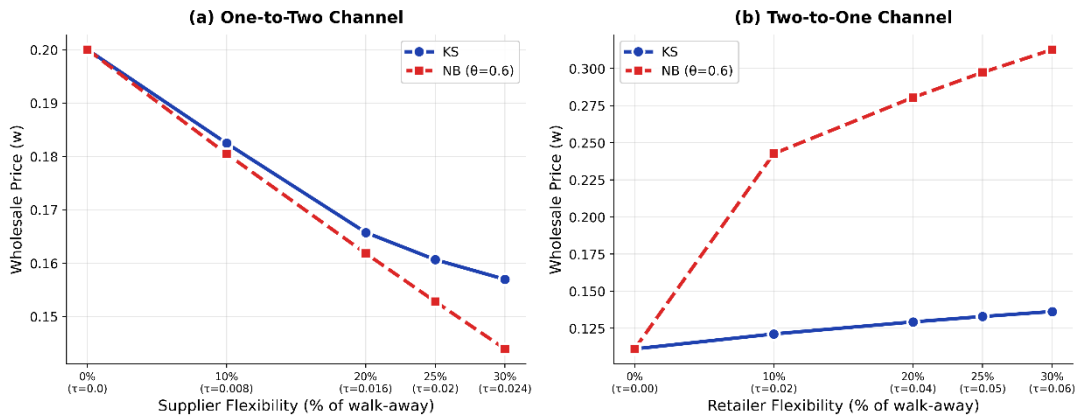
While KS responds with a bounded 20% price increase, NB nearly triples the wholesale price from 0.111 to 0.297. The ratio accelerates with flexibility (1.0  $\rightarrow$  2.2  $\rightarrow$  4.6  $\rightarrow$  8.5  $\rightarrow$  9.7), indicating NB’s response is convex while KS’s is approximately linear. Fig. 2 and Fig. 3 visualise these results.

**Figure 1: Flexibility Effects on Wholesale Price ( $\eta = 0.5$ )**



**Figure 2.** Flexibility effects on wholesale price ( $\eta = 0.5$ ). (a) One-to-two channel: modest KS–NB divergence, both solutions decline smoothly. (b) Two-to-one channel: dramatic divergence — KS increases by 20% while NB increases by 168%. The x-axis shows both the flexibility percentage and the corresponding  $\tau$  value

**Figure 2: Price Sensitivity to Flexibility ( $\eta = 0.5$ )**



**Figure 3.** Price sensitivity curves across the full flexibility range ( $\eta = 0.5$ ). (a) One-to-two channel: both solutions decline smoothly and similarly. (b) Two-to-one channel: KS responds approximately linearly while NB curves upward, reflecting an accelerating (convex) response to retailer flexibility signals

### 4.4. Why Does NB Explode? The Mathematical Explanation

The sensitivity difference stems from the fundamental mathematical structure of each solution. The NB solution maximises the multiplicative product:

$$N(w) = (\Pi - D^{eff})^{1-\theta} \cdot (\pi - d^{eff})^\theta \quad (13)$$

When  $d^{eff}$  decreases (retailer signals flexibility), the term

$(\pi - d^{eff})$  increases for any given  $\pi$ . Because the product is *multiplicative*, the first-order condition must rebalance, requiring a sharp upward shift in  $w$ . Each additional unit of flexibility further amplifies the previous increase — producing the observed convex, accelerating response.

The KS solution uses the proportionality condition:

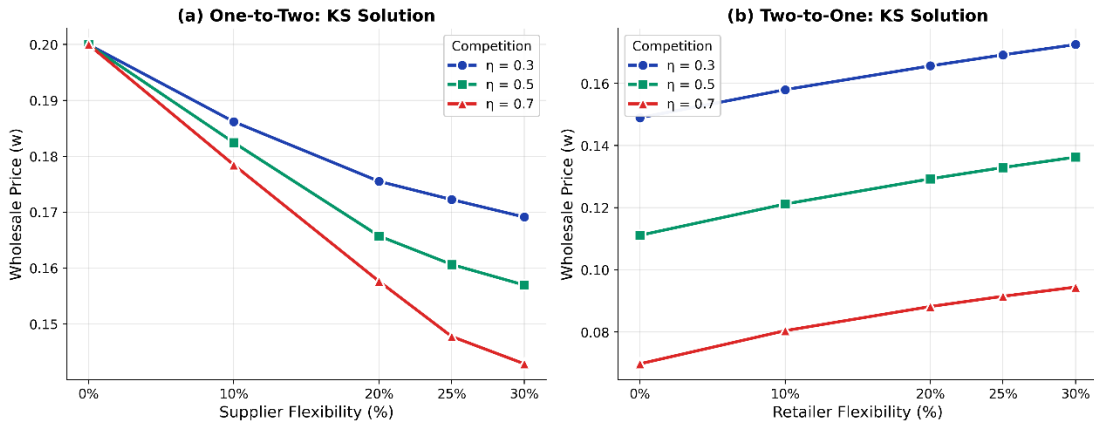
$$\frac{\Pi - D^{eff}}{\Pi - D^{eff}} = \frac{\pi - d^{eff}}{\pi - d^{eff}} \quad (14)$$

The utopia points  $\Pi$  and  $\bar{\pi}$  are anchored to the feasible set and do not shift when the disagreement point changes. When  $d^{eff}$  decreases, both the numerator and denominator on the right-hand side change in the same direction, partially cancelling each other. The KS solution is geometrically bounded and structurally incapable of producing explosive responses.

### 4.5. Competition Effects

Fig. 4 shows how competition intensity  $\eta$  modulates the

**Figure 3: Effect of Competition Level on KS Price Response**



**Figure 4.** Effect of competition intensity ( $\eta$ ) on the KS price response. (a) One-to-two channel: higher competition steepens the price decline. (b) Two-to-one channel: competition sets the baseline price level but slopes remain consistent across  $\eta$  values, confirming the robustness of the  $8.5\times$  sensitivity ratio

### 4.6. Efficiency Implications

A critical question is whether NB’s explosive response merely redistributes profits or destroys supply chain value. Table 5 and Fig. 5 provide the answer.

**Table 5.** Channel efficiency under flexibility ( $\eta = 0.5$ )

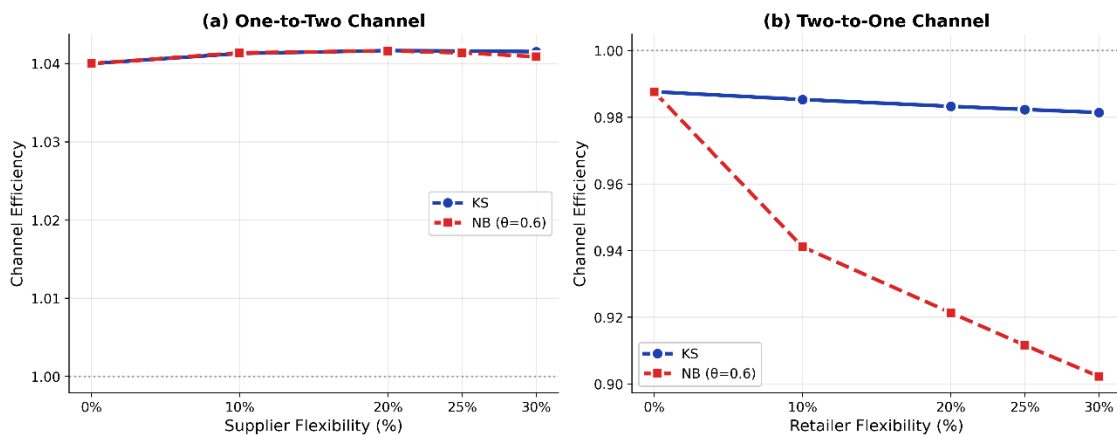
Channel	Solution	Efficiency at 0%	Efficiency at 30%	Change
One-to-Two	KS	104.0%	104.2%	+0.2 pp
One-to-Two	NB	104.0%	104.2%	+0.2 pp
Two-to-One	KS	98.9%	98.1%	-0.8 pp
Two-to-One	NB	98.9%	90.2%	-8.7 pp

flexibility response. In the one-to-two channel, higher  $\eta$  amplifies the price decline. In the two-to-one channel,  $\eta$  determines the baseline price level but does not materially alter the slope of the flexibility response. The  $8.5\times$  KS–NB sensitivity ratio persists across all competition levels tested ( $\eta \in \{0.3, 0.5, 0.7\}$ ), confirming it is a structural property of the solutions rather than an artefact of parameter choice.

Note: pp = percentage points. Efficiency > 100% occurs when decentralised profits exceed the integrated benchmark under the specific demand configuration.

In the two-to-one channel, KS efficiency falls only 0.8 pp, remaining above 98% even at 30% flexibility. NB efficiency collapses from 98.9% to 90.2% — an 8.7 pp loss representing approximately 9% of total supply chain value that is destroyed, not redistributed: NB’s high wholesale price propagates to higher retail prices, reduced consumer demand, lower sales volume, and deadweight loss that no party captures. In the one-to-two channel, both solutions maintain stable efficiency — the modest  $1.2\times$  divergence is insufficient to generate meaningful value destruction.

**Figure 4: Channel Efficiency Under Flexibility ( $\eta = 0.5$ )**



**Figure 5.** Channel efficiency under flexibility ( $\eta = 0.5$ ). (a) One-to-two channel: both solutions remain stable. (b) Two-to-one channel: KS maintains near-baseline efficiency throughout, while NB suffers an 8.7 percentage point loss at 30% flexibility — destroying approximately 9% of total supply chain value

Table 6 consolidates all key findings across both channels.

**Table 6.** Summary of key findings ( $\eta = 0.5$ )

Metric	One-to-Two	Two-to-One
Relevant flexibility	$\tau_S$ (Supplier)	$\tau_R$ (Retailer)
$\tau$ at 25% flexibility	0.020	0.049
KS price change at 25%	-19.7%	+19.6%
NB price change at 25%	-23.6%	+167.6%
NB/KS sensitivity ratio	1.2×	8.5×
KS efficiency at 30%	104.2%	98.1%
NB efficiency at 30%	104.2%	90.2%
Efficiency difference	0 pp	-8.7 pp

## 5. Discussion

### 5.1. The Stability Advantage of KS

Feng et al. [1] showed that the KS solution is “more reasonable” than NB at baseline. Our results extend this insight in a practically important direction: KS is also more stable under flexibility, and this stability directly preserves supply chain value. The 8.5× sensitivity ratio in the two-to-one channel represents an order-of-magnitude difference in how the two solutions respond to an identical flexibility signal. KS’s proportionality principle is anchored by utopia points that are invariant to disagreement point changes. NB’s multiplicative optimisation creates convex response surfaces that amplify any such shift.

### 5.2. Channel Asymmetry and the Flexibility Penalty

A striking asymmetry emerges across channels: 1.2× divergence in the one-to-two channel versus 8.5× in the two-to-one channel. This asymmetry reflects the relative magnitude of the monopolist’s walk-away value: the retailer’s walk-away ( $d^0 = 0.1975$ ) is more than twice the supplier’s ( $D^0 = 0.08$ ), providing substantially more leverage room for the Nash product to amplify flexibility signals.

Both solutions penalise the party that signals flexibility — an inherent feature of bargaining where disagreement points shape surplus allocation. But NB imposes a far harsher penalty. A retailer signalling 25% flexibility under NB dynamics faces a 168% price increase — far exceeding the concession offered. The same signal under KS produces only a 20% increase: proportional, bounded, and predictable. We term this phenomenon the flexibility penalty, and it is dramatically lower under KS in both channels.

### 5.3. Practical Implications

Understand your bargaining framework. Whether negotiations resemble KS-style proportionality or NB-style power-weighted optimisation has consequential implications for what happens when flexibility is signalled. Firms in environments where NB dynamics dominate should exercise substantially greater caution before communicating flexibility.

Signalling flexibility is risky, especially for monopolist retailers. A monopoly retailer facing competing suppliers might assume its structural position provides protection. Our results show the opposite: this position paradoxically makes flexibility more dangerous, as suppliers have significant room to exploit any walk-away concession under NB-style bargaining.

KS-style negotiation protocols provide structural protection. Organisations that can shape the negotiation

framework — through contractual provisions, third-party mediation, or institutional norms — should advocate for proportionality-based rules when flexibility signals are likely to be exchanged. Such protocols preserve supply chain value that would otherwise be destroyed.

Channel structure determines flexibility strategy. A 25% signal in the one-to-two channel results in modest price reductions under both solutions; the same signal in the two-to-one channel produces a measured KS response but a catastrophic NB response. Supply chain managers must account for their channel position when deciding whether, and how much, to signal flexibility.

## 5.4. Theoretical Contributions and Limitations

This paper contributes to bargaining theory by demonstrating that KS and NB carry distinct consequences beyond the baseline case. Allowing disagreement points to vary — even exogenously — reveals a fundamental structural difference invisible at baseline. We further contribute methodologically through the percentage-based flexibility formulation, which is essential for reliable cross-channel comparison.

Several limitations bound the analysis. Flexibility is modelled as an exogenous parameter rather than a strategic choice; endogenising it would reveal equilibrium signalling strategies. We focus on simultaneous bargaining and symmetric firms; sequential bargaining and firm asymmetry are natural extensions. Empirical validation with experimental or field data would substantially strengthen the practical relevance of these results.

## 6. Summary

This paper introduces flexible disagreement points into the supply chain bargaining framework of Feng et al. [1] and analyses how the Kalai-Smorodinsky and Nash Bargaining solutions respond differently to flexibility signals. Four findings emerge from a focused computational study of 60 scenarios across two channel structures and competition levels  $\eta \in \{0.3, 0.5, 0.7\}$ .

At baseline, both solutions produce identical outcomes at  $\theta = 0.6$ , confirming Feng et al.’s characterisation. Under flexibility, NB is 1.2× more reactive than KS in the one-to-two channel and 8.5× more reactive in the two-to-one channel. NB’s hyper-reactivity destroys approximately 9% of total supply chain value in the two-to-one channel while KS maintains near-baseline efficiency. These results are robust across all competition levels and flexibility ranges examined.

The central contribution: Feng et al. showed KS is “more reasonable” at baseline. We show it is also “more stable” under flexibility — and this stability preserves real supply chain value. KS’s proportionality architecture bounds sensitivity to flexibility signals, whereas NB’s multiplicative structure can produce explosive reactions destroying nearly 10% of channel value. For both researchers and practitioners, the choice of bargaining framework matters not only at the negotiating table but in how that framework responds to the signals that parties inevitably send.

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